Example 10: Consider the matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ .

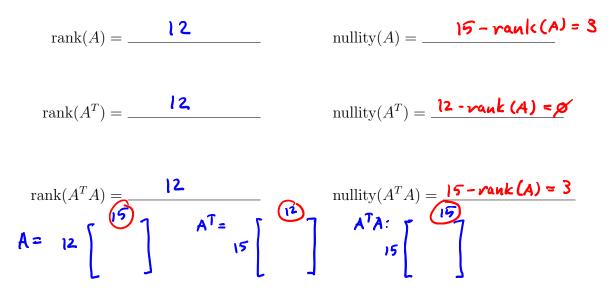
1. Calculate  $\operatorname{rank}(A)$ .

2. Calculate rank $(A^T)$ . What do you observe?

Theorem 5: Let A be a  $m \times n$  matrix. Then

 $\operatorname{rank}(A) = \operatorname{rank}(A^T)$ 

*Example 11:* <u>Fill in the Blanks</u>: Let A be a  $12 \times 15$  matrix. If  $A\mathbf{x} = \mathbf{b}$  has a solution for all **b**, then



*Example 12:* <u>Fill in the Blanks</u>: Let A be a  $17 \times 11$  matrix. If  $\underline{A}^T A$  is invertible, then

